

LESSON 2-1

Parallel Lines



Lesson Overview

KEY CONCEPTS

Mathematics Objectives

- ✓ Define parallel lines using the undefined terms *point* and *line*.
- ✓ Prove theorems about lines and angles and use them to find the measures of angles formed by parallel lines and a transversal.

Language Objective

- ✓ Use correct terms for angles formed by parallel lines and a transversal.

Essential Understanding

When parallel lines are cut by a transversal, the special angle pairs that are formed are congruent, supplementary, or both.

COHERENCE

Previously in this course, students:

- Studied special angle pairs, including adjacent, supplementary, and vertical angles and linear pairs.

In this lesson, students:

- Define angle pairs formed by parallel lines and a transversal and prove theorems about them.
- Use the Same-Side Interior Angles Postulate and the Alternate Interior Angles, Corresponding Angles, and Alternate Exterior Angles Theorems to find the measures of angles formed by parallel lines and a transversal.

Later in this topic, students will:

- Use congruent and supplementary special angle pairs to prove lines parallel.

BALANCE

This lesson emphasizes a blend of *conceptual understanding* and *procedural skill and fluency*.

- Students use deductive reasoning with stated postulates, definitions, and proven theorems to prove theorems about special angle pairs associated with parallel lines.
- Students use postulates and theorems to find the measures of special angle pairs associated with parallel lines.



Presentation is available at SavvasRealize.com.

Math Talk

For use before this lesson.

Every Picture Tells a Story

Lesson 2-1

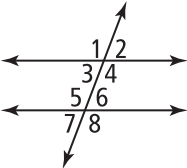
What pairs of equal angles can you find?
What term describes the pair of equal angles?



TEKS G.5A

Every Picture Tells a Story Present the illustration, providing time for students to look it over and see how the streets are named. Ask students to identify an intersection that appears to make a pair of equivalent angles. Then ask for volunteers to share the intersection and angles they identified. Show the simplified image below.

Have students sort the numbered angles into larger angles and smaller angles.



Skills Review & Practice

Present or assign to reactivate prior knowledge.

- **G12** Properties of Parallel Lines

Vocabulary Builder

Use the Vocabulary Builder activity online to support understanding of the terms *alternate exterior angles*, *alternate interior angles*, *corresponding angles*, *parallel*, and *transversal*.



Texas Essential Knowledge and Skills (TEKS)

Linear functions, equations, and inequalities. The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:

G.5A solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides. Also **G.6A**

Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

G.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.



English Language Proficiency Standards (ELPS)

SPEAKING 2B Speak using content-area vocabulary during formal and informal classroom interactions to demonstrate acquisition of new words and high-frequency words.



College and Career Readiness Standards

II.C.3 Recognize and use algebraic properties, concepts, and algorithms to solve equations, inequalities, and systems of linear equations and inequalities. Also **III.C.1** and **III.D.3**

Standards in **VII** Problem Solving and Reasoning, **VIII** Communication and Representation, and **IX** Connections are infused throughout the program.



EXPLORE & SHARE

INSTRUCTIONAL FOCUS Students become familiar with the angles formed by parallel lines and a transversal and look for relationships between these angles.

STUDENT COMPANION Students can do their work for the task on their *Student Companion* pages, which you can print from SavvasRealize.com.

Before WHOLE CLASS

Implement Tasks that Promote Reasoning and Problem Solving

- Q:** What are parallel lines? [coplanar lines that do not intersect]
- Q:** The parallel lines are “cut by a transversal.” What do you think is meant by the term “cut”? What do you think a transversal is? [Sample: intersected by; a line that intersects two parallel lines]
- Q:** Can you think of a real-world situation that involves parallel lines and a transversal? [Sample: Two sides of a street with a crosswalk between them.]

During SMALL GROUP

Support Productive Struggle in Learning Mathematics

- Q:** Do you see any types of special angles in the diagram? [Yes, there are linear pairs and pairs of vertical angles.]
- Q:** Without knowing the measures of the angles, what relationships do you think *might* exist between certain pairs of angles? [Some will be congruent, such as the vertical angles. Some will be supplementary, such as the linear pairs.]
- Q:** How can you verify whether your conjectures are true? [Use a protractor to measure the angles.]

For Early Finishers

- Q:** In the diagram, $\angle 2$ and $\angle 5$ are on the same side of the transversal and each is above a parallel line. Describe the positions of the following angles with respect to the transversal and the parallel lines: $\angle 3$ and $\angle 6$; $\angle 1$ and $\angle 7$; $\angle 3$ and $\angle 5$. [opposite sides of the transversal, inside the parallel lines; opposite sides of the transversal, outside the parallel lines; same side of the transversal, inside the parallel lines]

After WHOLE CLASS

Facilitate Meaningful Mathematical Discourse

- Ask students to share with the class what they wrote in Part B.
- Q:** How did you decide if the same relationships between the angles would still be true with another transversal? [Sample: Draw another transversal and measure the new angles formed. Repeat with several more transversals.]



Explore & Share is available at SavvasRealize.com.

STUDENT EDITION

2-1

Parallel Lines

I CAN... determine the measures of the angles formed when parallel lines are intersected by a transversal.

TEKS
G.5A Investigate patterns to make conjectures about geometric relationships, including angles formed by parallel lines cut by a transversal, criteria required for triangle congruence, special segments of triangles, diagonals of quadrilaterals.

EXPLORE & SHARE

The diagram shows two parallel lines cut by a transversal.

A. **Analyze Math Relationships** What relationships among the measures of the angles do you see?

B. Suppose a different transversal intersects the parallel lines. Would you expect to find the same relationships with the measures of those angles? Explain.

ESSENTIAL QUESTION

What angle relationships are created when parallel lines are intersected by a transversal?

EXAMPLE 1

Identify Angle Pairs

SAMPLE RESPONSES

- A. The measure of each angle is either 58° or 122° , so each pair of angles is either congruent or supplementary. The angles that form linear pairs are supplements and the vertical angles are congruent.
- B. Yes; the measures of the angles might change because the slope of the new transversal might be different than the original one, but the pairs of angles should still be congruent, supplementary, or both.

PROCESS STANDARDS CONNECTION Use with EXPLORE & SHARE

Analyze Math Relationships What theorems have you already learned that can be used to show why some of the angles formed are congruent?

[Sample: The Vertical Angles Theorem explains why vertical angles are congruent.]



Activity

INTRODUCE THE ESSENTIAL QUESTION

Establish Mathematics Goals to Establish Learning

Students will learn the formal names for the types of angles formed when parallel lines are intersected by a transversal and prove that these angles are congruent, supplementary, or both.

EXAMPLE 1 Identify Angle Pairs

Pose Purposeful Questions

- Q:** How can you use the terms *same-side* and *exterior* to help you identify pairs of same-side exterior angles? [Sample: Look for two angles that are on the *same side* of the transversal and on the *exterior* of, or outside, the horizontal lines.]
- Q:** What do you notice about the numbers of alternate interior angles, same-side exterior angles, and corresponding angles? [There are two pairs of alternate interior and same-side exterior angles and four pairs of corresponding angles.]

Try It! Answers

1. a. $\angle 4$ and $\angle 6$: same-side interior angles; $\angle 4$ and $\angle 5$: alternate interior angles; $\angle 4$ and $\angle 8$: corresponding angles
- b. $\angle 7$ and $\angle 2$: same-side exterior angles; $\angle 7$ and $\angle 1$: alternate exterior angles; $\angle 7$ and $\angle 3$: corresponding angles



Examples are available at SavvasRealize.com.

geometric relationship, including angles formed by parallel lines cut by a transversal, criteria required for triangle congruence, special segments of triangles, diagonals of quadrilaterals, interior and exterior angles of polygons, and special segments and angles of circles choosing from a variety of tools. Also G.6A

Mathematical Process Standards
G.1B, G.1D, G.1F

BUILD G.R.I.T. Time Management
Reinforce the different pairs of angles created from a transversal. Create visual notes for each pair, including where the pairs of lines are not parallel.

ESSENTIAL QUESTION What angle relationships are created when parallel lines are intersected by a transversal?

EXAMPLE 1 Identify Angle Pairs

Identify the pairs of angles of each angle type made by the snowmobile tracks.

Try It! 1. Which angle pairs include the named angle?
a. $\angle 4$ b. $\angle 7$

Language Routine: Co-Craft Questions and Problems

After reading Example 1, invite students to develop their own questions and problems about pairs of equal angles. The problems can take either of two forms.

1. Name a pair of equal angles, using the numbers on the diagram in Example 1. Ask other students to say what kind of angle pair the two angles form: corresponding, alternate interior, or alternate exterior.
2. Name a kind of angle pair and ask other students to point to or name two angles that would fit the definition.

EXIT TICKET

Use this Exit Ticket anytime after Example 2.

ITEM	TEKS	DOK
1	G.6A	2
2	G.5A	2

Use the Exit Ticket results to inform instructional decisions.

Instructional Implications

In Question 1, if students chose answer choice A, they may NOT have a clear understanding that the angles were on the same side of the transversal but misjudged that both angles were interior.

In Question 2, if students chose answer choice or C, they may NOT have a clear understanding that angles 2 and 5 were congruent or complementary. If they chose answer choice D, they may have thought the question called for the sum of angles 2 and 5.

As your instruction in parallel lines continues, plan to address common errors and misconceptions.

- Remind students that alternate exterior angles are outside the parallel lines, while alternate interior angles are between the parallel lines. Use Do You Know How item 5 or 6, if needed.



Formative Assessment The Exit Ticket is available at Savvas Realize® and in the Assessment Sourcebook.

Name Use this Exit Ticket anytime after Example 2. **enVision**
SavvasRealize.com

2-1 Exit Ticket
Parallel Lines

1. Given $m \parallel n$, choose the postulate or theorem that justifies why $\angle 8 \cong \angle 4$.

☐ A Same-Side Interior Angles Postulate
☐ B Alternate Interior Angles Theorem
☒ C Corresponding Angles Theorem
☐ D Alternate Exterior Angles Theorem

2. Given $m \parallel n$, determine $m\angle 2$ if $m\angle 5 = 125^\circ$.

- Remind them that a transversal cutting at an angle through two parallel lines creates four acute angles and four obtuse angles and that all the acute angles and all the obtuse angles have the same measure.



Activity

EXAMPLE 2 Explore Angle Relationships

Build Procedural Fluency from Conceptual Understanding

Q: In the diagram for Postulate 2-1, do you see any other same-side interior angles?

[Yes, there is a pair on the opposite side of the transversal from $\angle 1$ and $\angle 2$.]

Q: What types of related angles do you see in the diagram that might help you find the measure of each angle?

[vertical angles, linear pairs, and same-side interior angles]

Q: What angle forms vertical angles with the given angle? What is its measure?

[$\angle 7$; x° ; vertical angles are congruent.]

Q: What angles form a linear pair with the given angle? How can you find their measures?

[$\angle 6$ and $\angle 8$; subtract x° from 180° ; angles that form a linear pair are supplementary.]

Q: What angle forms a pair of same-side interior angles with the given angle? How can you find its measure?

[$\angle 2$; subtract x° from 180° ; since the lines are parallel, the same-side interior angles are supplementary.]

Q: How can you find the measures of the remaining angles?

[Each remaining angle is congruent to either the given angle or to $\angle 2$. You need to determine which of these each angle is congruent to, and then you will have found the measure of each angle.]

Try It! Answers

- 118 or 62
- 66

Elicit and Use Evidence of Student Thinking

Q: What do you know about the measure of each angle in the figure?

[Each angle is either congruent to or supplementary to $\angle 4$.]

Q: What angles are congruent to $\angle 4$? What angles are supplementary to $\angle 4$?

[$\angle 2$, $\angle 6$, $\angle 8$; $\angle 1$, $\angle 3$, $\angle 5$, $\angle 7$]

Common Error

Try It! 2 Students may have difficulty getting started because of the orientation of the parallel lines. If this is the case, have students rotate their books until the parallel lines appear horizontal. Remind students that, in general, the orientation of a figure does not affect the strategy they should use when solving a problem.



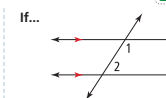
Postulate 2-1 and Theorem 2-1 are Powered by Desmos. Examples are available at SavvasRealize.com.

CONCEPTUAL UNDERSTANDING

ANALYZE MATH RELATIONSHIPS
What patterns do you notice about the angles formed by two parallel lines cut by a transversal?

POSTULATE 2-1 Same-Side Interior Angles Postulate

If a transversal intersects two parallel lines, then same-side interior angles are supplementary.



Then... $m\angle 1 + m\angle 2 = 180$

EXAMPLE 2 Explore Angle Relationships

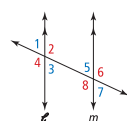
How can you express each of the numbered angles in terms of x ?

By Postulate 2-1, $\angle 2$ and the angle with measure x are supplementary.
The angle with measure x and $\angle 7$ are vertical angles. Both $\angle 6$ and $\angle 8$ form a linear pair with the angle with measure x .

You can make conclusions about $\angle 1$, $\angle 3$, and $\angle 4$ as you did with $\angle 6$, $\angle 7$, and $\angle 8$.
The angles with measures equal to x are $\angle 1$, $\angle 3$, and $\angle 7$.
The angles that are supplementary to the angle with measure x are $\angle 2$, $\angle 4$, $\angle 6$, and $\angle 8$, which have measure $(180 - x)$.

Try It! 2. Use the figure at the right.

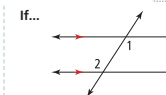
- If $m\angle 4 = 118$, what is the measure of each of the other angles?
- If $m\angle 2 = 9x - 12$ and $m\angle 4 = 6x + 30$, what is $m\angle 5$?



THEOREM 2-1 Alternate Interior Angles Theorem

If a transversal intersects two parallel lines, then alternate interior angles are congruent.

PROOF: SEE EXAMPLE 3.



Then... $\angle 1 \cong \angle 2$



Activity

EXAMPLE 3 Prove the Alternate Interior Angles Theorem

Implement Tasks that Promote Reasoning and Problem Solving

- Q:** In the diagram for Example 3, why do you think $\angle 3$ is labeled?
[Sample: Any angle that might help with the proof can be labeled.]
- Q:** How is $\angle 3$ related to $\angle 1$? How is $\angle 3$ related to $\angle 2$?
[$\angle 3$ and $\angle 1$ are same-side interior angles; $\angle 3$ and $\angle 2$ form a linear pair.]
- Q:** What is unique about the location of $\angle 3$? Can the same be said for any other angles in the diagram?
[Sample: It is supplementary to $\angle 1$ and to $\angle 2$. The angle that forms a pair of same-side interior angles with $\angle 2$ is also supplementary to $\angle 1$ and to $\angle 2$.]

Try It! Answer

3. You are given $m \parallel n$. By the Angle Addition Postulate, $m\angle 1 + m\angle 3 = 180$, and by the Same-Side Interior Angles Postulate, $m\angle 2 + m\angle 3 = 180$. So $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$ by the Transitive Property of Equality. By the Subtraction Property of Equality, $m\angle 1 = m\angle 2$, so $\angle 1 \cong \angle 2$ by the definition of congruence.

Elicit and Use Evidence of Student Thinking

- Q:** In the Try It!, how can you use $\angle 3$ to help you prove the Corresponding Angles Theorem?
[You can use its relationships to $\angle 1$ and $\angle 2$. $\angle 3$ and $\angle 1$ form a linear pair and by that definition are supplementary; $\angle 3$ and $\angle 2$ are same-side interior angles and by Postulate 2-1 are supplementary.]

PROCESS STANDARDS CONNECTION

Use with EXAMPLES 1, 2, & 3

Apply Math Suppose that a transversal intersects a pair of parallel lines and one of the angles created measures x . What must be true of the other interior angles that are formed?

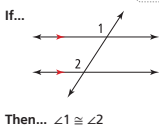
[They must measure x or $(180 - x)$.]



Theorems 2-2 and 2-3 are Powered by Desmos.
Examples are available at SavvasRealize.com.

THEOREM 2-2 Corresponding Angles Theorem

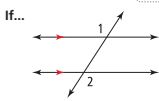
If a transversal intersects two parallel lines, then corresponding angles are congruent.
PROOF: SEE EXAMPLE 3 TRY IT.



Then... $\angle 1 \cong \angle 2$

THEOREM 2-3 Alternate Exterior Angles Theorem

If a transversal intersects two parallel lines, then alternate exterior angles are congruent.
PROOF: SEE EXERCISE 12.



Then... $\angle 1 \cong \angle 2$

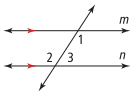
PROOF

EXAMPLE 3 Prove the Alternate Interior Angles Theorem

Prove the Alternate Interior Angles Theorem.

Given: $m \parallel n$

Prove: $\angle 1 \cong \angle 2$



Plan: Use the Same-Side Interior Angles Postulate to show $\angle 1$ is supplementary to $\angle 3$. Then show that angles 1 and 2 are congruent because they are both supplementary to the same angle.

Proof:

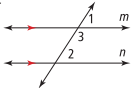
Statements	Reasons
1) $m \parallel n$	1) Given
2) $\angle 1$ and $\angle 3$ are supplementary	2) Same-Side Interior $\angle$ s Postulate
3) $m\angle 1 + m\angle 3 = 180$	3) Def. of supplementary angles
4) $m\angle 2 + m\angle 3 = 180$	4) Angle Addition Postulate
5) $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	5) Transitive Property of Equality
6) $m\angle 1 = m\angle 2$	6) Subtraction Property of Equality
7) $\angle 1 \cong \angle 2$	7) Def. of congruence

COMMON ERROR
Remember that for the proof to be complete, the last statement of the proof must match what you are trying to prove.

Try It! 3. Prove the Corresponding Angles Theorem.

Given: $m \parallel n$

Prove: $\angle 1 \cong \angle 2$



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Targeted ELPS Support SPEAKING 2B

PRE-PRODUCTION Point to the term *alternate* in the Example title. Explain that *alternate* can be a verb, meaning to act or do by turns. Here, alternate is an adjective, meaning something related to every other one in a series. Read the example text and have students give a thumbs up each time you say the word *alternate*.

BEGINNING Point out the term *alternate* in the Example title. Explain that *alternate* can be a verb, meaning to act or do by turns. It can also be a noun, meaning a person acting in the place of another or a substitute. Here, alternate is an adjective, meaning something related to every other one in a series. Read the example text. Have students point to the two angles the proof will prove as alternate interior angles (1 and 2).

INTERMEDIATE Discuss the prefix *inter-*, which means inside. Point out that the prefix *exter-* means outside. Explain why the terms *interior* and *exterior* are used in relation to parallel lines. Ask: **Why do the interior and exterior parts of the theorem names refer to parallel lines and not transversal lines?** [Two lines are needed to create a space inside and a space outside.]

HIGH INTERMEDIATE Have students work in small groups to discuss the prefix *inter-* (which means inside) and the prefix *exter-* (which means outside). Ask students why the terms *interior* and *exterior* are used in relation to parallel lines. Ask: **What do inter- and exter- appear to mean?** [inside and outside] Ask: **Why do the interior and exterior parts of the theorem names refer to parallel lines and not transversal lines?** [Two lines are needed to create a space inside and a space outside.]

ADVANCED Have students review the meanings of the words *congruent* and *supplementary*. Then, have students work in pairs. One student uses the numbers to name any two angles in the Example 3 diagram. Then, the other student says whether the two angles are congruent or supplementary and if congruent what kind of pair they are.

(Use with EXAMPLE 3)



Activity

EXAMPLE 4 Use Parallel Lines to Prove an Angle Relationship

Support Productive Struggle in Learning Mathematics

Q: You are given $\overline{AC} \parallel \overline{DF}$ and $\overline{BE} \perp \overline{DF}$. What conclusions can you draw from these pieces of information?

[Same-side interior angles are supplementary; alternate interior angles, corresponding angles, and alternate exterior angles are congruent. $\angle BED$ and $\angle BEF$ are right angles, so $m\angle BED = m\angle BEF = 90^\circ$.]

Q: Explain the placement of Statement 5

[Statement 5 is the given information, which is usually listed as the first statement of a proof. It is placed here because it is used in combination with Statement 4 to produce Statement 6.]

Try It! Answer

4. $m\angle 1 + m\angle ACD = 180$ by the Same-Side Interior Angles Post., and $m\angle ACD = m\angle 3 + m\angle 4$ by the Angle Add. Post. By the Subst. Prop. of Equ., $m\angle 1 + m\angle 3 + m\angle 4 = 180$. $\angle 2 \cong \angle 4$ by the Alt. Int. Angles Thm., so $m\angle 2 = m\angle 4$ by the definition of congruence. By the Subst. Prop. of Equ., $m\angle 1 + m\angle 3 + m\angle 2 = 180$, so $m\angle 1 + m\angle 2 + m\angle 3 = 180$ by the Comm. Prop. of Add.

EXAMPLE 5 Find Angle Measures

Elicit and Use Evidence of Student Thinking

Q: What strategy might help you solve this type of problem?

[Sample: Redraw only the intersecting lines from the diagram, and label the lines and angles with the given information.]

Try It! Answer

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PROCESS STANDARDS CONNECTION

Use with
EXAMPLES 4 & 5

Connect Math Ideas What are some strategies you can use to find unknown angle measures?

[Sample: Look for parallel lines and transversals, and find special angle pairs; look for linear pairs; look for vertical angles.]

Support Student Understanding

USE WITH EXAMPLE 4 Struggling students may have difficulty understanding the presented proof.

- Walk through the proof, step by step, with students. Refer to the diagram, and ask the following questions.

Q: Look at Statement 3. How do you know $m\angle BED = m\angle BEF = 90^\circ$? [It follows from the given information $\overline{BE} \perp \overline{DF}$.]

Q: Look at Statement 6. How many pairs of congruent alternate interior angles are there, and why are only these two pairs listed? [Three transversals intersect two parallel lines, forming six pairs of alternate interior angles. These two pairs are listed because they help connect $\angle BAE$ to $\angle BCE$.]



Examples are available at SavvasRealize.com.

PROOF

EXAMPLE 4 Use Parallel Lines to Prove an Angle Relationship

Use the diagram to prove the angle relationship.

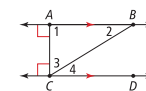
Given: $\overline{AC} \parallel \overline{DF}$, and $\overline{BE} \perp \overline{DF}$, $\angle AEB \cong \angle CEB$

Prove: $\angle BAE \cong \angle BCE$

Proof:

Statements	Reasons
1) $\overline{AC} \parallel \overline{DF}$, $\overline{BE} \perp \overline{DF}$	1) Given
2) $\angle BED$, $\angle BEF$ are rt. angles	2) Def. of perpendicular
3) $m\angle BED = m\angle BEF = 90$	3) Def. of rt. angles
4) $m\angle AED + m\angle AEB = 90$, $m\angle CEF + m\angle CEB = 90$	4) Angle Addition Postulate
5) $\angle AEB \cong \angle CEB$	5) Given
6) $\angle AED \cong \angle CEF$	6) Congruent Complements Thm.
7) $\angle BAE \cong \angle AED$, $\angle BCE \cong \angle CEF$	7) Alt. Interior $\angle$ s Thm.
8) $\angle BAE \cong \angle BCE$	8) Transitive Prop. of Congruence

Try It! 4. Given $\overline{AB} \parallel \overline{CD}$, prove that $m\angle 1 + m\angle 2 + m\angle 3 = 180$.



APPLICATION

EXAMPLE 5 Find Angle Measures

The white trim shown for the wall of a barn should be constructed so that $\overline{AC} \parallel \overline{EG}$, $\overline{JA} \parallel \overline{HB}$, and $\overline{JC} \parallel \overline{KG}$. What should $m\angle 1$ and $m\angle 3$ be?

You can create a simple drawing of the wall of the barn so that the angles are easier to visualize. Look for relationships among the angles.

By the Same-Side Interior Angles Postulate,
 $m\angle 1 + 68 = 180$.

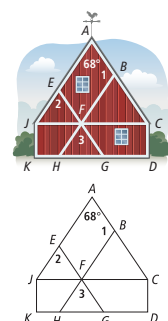
$$m\angle 1 = 180 - 68 = 112$$

By the Corresponding Angles Theorem,
 $\angle EAB \cong \angle 2$ and $\angle 2 \cong \angle 3$, so $\angle 3 \cong \angle EAB$ by the Transitive Property of Congruence.

$$m\angle 3 = 68$$

So, $m\angle 1 = 112$ and $m\angle 3 = 68$.

Try It! 5. If $m\angle EGF = 56$, find $m\angle FHK$.



Extend Student Thinking

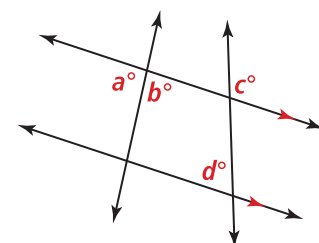
USE WITH EXAMPLE 5 Have advanced students find the measures of angles formed by parallel lines and transversals.

Q: What is the value of x if $a = x + 20$ and $b = x + 10$?

[75; The angles form a linear pair.]

Q: What is the value of y if $c = y$ and $d = y - 40$?

[110; The angles are supplements.]





CONCEPT SUMMARY Parallel Lines and Angle Pairs

Pose Purposeful Questions

- Q: When two parallel lines are cut by a transversal, how many pairs of same-side interior angles are formed? Alternate interior angles? Corresponding angles? Alternate exterior angles? [2; 2; 4; 2]
- Q: When two parallel lines are cut by a transversal, how many angles are formed? How many different angle measures are there? Explain.
[8; 2, unless the transversal is perpendicular to the parallel lines, in which case there is only one angle measure, 90.]
- Q: In your own words, summarize the angle relationships that exist when two parallel lines are cut by a transversal.
[Sample: If two parallel lines are cut by a transversal, then the pairs of same-side interior angles are supplementary, and the pairs of alternate interior angles, corresponding angles, and alternate exterior angles are congruent.]

Talk About Math Ideas

Encourage students to explore various angles created by a transversal and parallel lines. Have them distinguish differences and make note of them in practice with others.

- Q: Think about the situation you discussed during the Explore & Share. What theorems could you use to identify angles in the figure which are supplementary or congruent?
[The Same-Side Interior Angles Postulate identifies supplementary angles. The Alternate Exterior Angles, Alternate Exterior Angles, and Corresponding angles Theorems identify congruent angle pairs.]

Do You UNDERSTAND? | Do You KNOW HOW?

Common Error

Exercise 8 Because students did not learn a Same-Side Exterior Angles Theorem, they may have difficulty finding the measure of $\angle 7$. Encourage these students to use a combination of definitions and theorems. For example, $\angle 1 \cong \angle 8$ by the Alternate Exterior Angles Theorem, and $\angle 8$ and $\angle 7$ are supplementary because they form a linear pair. This should help students understand that $\angle 1$ and $\angle 7$ are also supplementary.

Answers

- Corresponding, alternate interior, and alternate exterior angles are congruent. Same-side interior and same-side exterior angles are supplementary. The two types of congruent angles formed are supplementary to each other.
- alternate interior angles, alternate exterior angles, corresponding angles, and vertical angles
- Sample: Leah cannot use the theorem because it is not indicated that the lines intersected by the transversal are parallel.

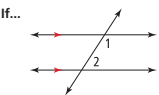


Concept Summary, Do You Understand, and Do You Know How are available at SavvasRealize.com.

CONCEPT SUMMARY Parallel Lines and Angle Pairs

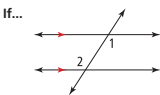
There are four special angle relationships formed when parallel lines are intersected by a transversal.

POSTULATE 2-1 Same-Side Interior Angles Postulate



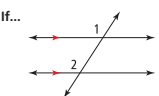
Then... $m\angle 1 + m\angle 2 = 180$

THEOREM 2-1 Alternate Interior Angles Theorem



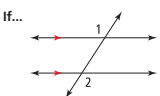
Then... $\angle 1 \cong \angle 2$

THEOREM 2-2 Corresponding Angles Theorem



Then... $\angle 1 \cong \angle 2$

THEOREM 2-3 Alternate Exterior Angles Theorem

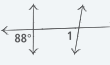


Then... $\angle 1 \cong \angle 2$

Talk About Math Ideas What relationships exist between vertical angles and the angles formed between parallel lines and a transversal?

Do You UNDERSTAND?

- ESSENTIAL QUESTION** What angle relationships are created when parallel lines are intersected by a transversal?
- Vocabulary** When a transversal intersects two parallel lines, which angle pairs are congruent?
- Error Analysis** What error did Leah make?



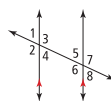
$m\angle 1 = 88$ by Corresponding Angles Theorem

- Analyze Math Relationships** For any pair of angles formed by a transversal intersecting parallel lines, what are two possible relationships?

Do You KNOW HOW?

Use the diagram for Exercises 5–10.

Classify each pair of angles. Compare angle measures, and give the postulate or theorem that justifies it.



5. $\angle 2$ and $\angle 6$

6. $\angle 3$ and $\angle 5$

If $m\angle 1 = 71$, find the measure of each angle.

7. $\angle 5$

8. $\angle 7$

If $m\angle 3 = 3x + 12$ and $m\angle 5 = 2x + 3$, find the measure of each angle.

9. $\angle 5$

10. $\angle 7$

11. Elm St. and Spruce St. are parallel. What is $m\angle 1$?



LESSON 2-1 Parallel Lines 75

- The angles can be congruent or supplementary.
- congruent, Corresponding Angles Theorem
- supplementary, Same-Side Interior Angles Postulate
- 71°
- 109°
- 69°
- 111°
- 68°



Practice

PRACTICE & PROBLEM SOLVING



Practice & Problem Solving, Adaptive Practice and Spiral Review are available at SavvasRealize.com.

Space and Interleaved Practice You may opt to assign the automatically scored lesson digital Practice and Problem Solving or Spiral Review.

Assignment Guide

On-Level	Advanced
12, 14, 15, 17–26, 28–31	12–16, 20–31

Engage Through Student Choice

Go online for an assignment strategy.

Item Analysis

Example	Items	DOK	TEKS	
			Content	Process
1	17–19	1	G.4B	
	27	2	G.4B	G.1D
2	15	2	G.4B	G.1B
	20, 28–30, 31	2	G.4B	
	14	3	G.4B	
3	21, 22	2	G.4B	
	12	3	G.4B	G.1D
4	23	3	G.4B	
5	16	2	G.4B	G.1F
	24	2	G.4B	
	13	3	G.4B	
	25, 26	3	G.4B	G.1B

Answers

12. Sample:

Statements	Reasons
$m \parallel n$	Given
$\angle 1 \cong \angle 3$	Corresponding Angles Theorem
$\angle 3 \cong \angle 2$	Vertical Angles Theorem
$\angle 1 \cong \angle 2$	Transitive Property of Congruence

13. $\angle 1$ and $\angle 3$ are supplementary and $\angle 2$ and $\angle 4$ are supplementary because they are same-side interior angles.

14. While there is no Same-Side Exterior Angles Theorem given, same-side exterior angles are supplementary, not congruent. $m\angle 1 = 180 - 72 = 108$

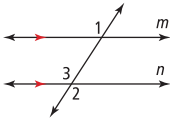
PRACTICE & PROBLEM SOLVING

UNDERSTAND

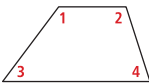
12. **Communicate Math Ideas** Write a two-column proof of the Alternate Exterior Angles Theorem.

Given: $m \parallel n$

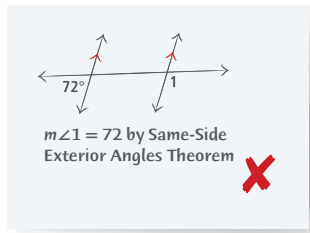
Prove: $\angle 1 \cong \angle 2$



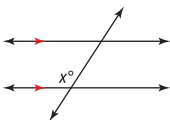
13. **Higher Order Thinking** Using what you know about angle pairs formed by parallel lines and a transversal, how are $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ related in the trapezoid? Explain.



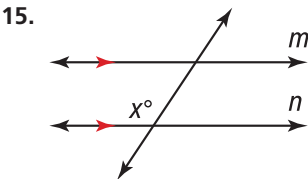
14. **Error Analysis** What error did Tyler make?



15. **Use a Problem-Solving Model** In the diagram shown, if $x + y = 180$, label the remaining angles as x° or y° .



16. **Mathematical Connections** A transversal intersects two parallel lines. The measures of a pair of alternate interior angles are $5v$ and $2w$. The measures of a pair of same-side exterior angles are $10w$ and $5v$. What are the values of w and v ?



16. $w = 15, v = 6$

17. Sample: $\angle 4$ and $\angle 7$

18. Sample: $\angle 7$ and $\angle 3$

PRACTICE

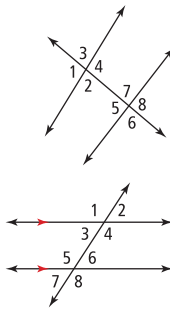
Identify a pair of angles for each type. SEE EXAMPLE 1

17. same-side interior

18. corresponding

19. alternate exterior

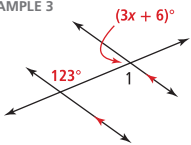
20. Which angles are supplementary to $\angle 1$? Which are congruent to $\angle 1$? SEE EXAMPLE 2



Find each measure. SEE EXAMPLE 3

21. $m\angle 1$

22. x



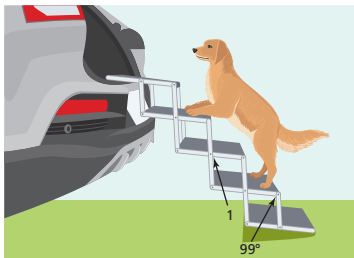
23. Opposite sides of a parallelogram are parallel. Prove that opposite angles of a parallelogram are congruent. SEE EXAMPLE 4

Given: $ABCD$ is a parallelogram.

Prove: $\angle A \cong \angle C, \angle B \cong \angle D$



24. For safety and strength, the supports of a set of scissor steps form parallelograms. What should the measure of $\angle 1$ be to allow a dog to climb safely into its owner's car? SEE EXAMPLE 5



See answers for Problems 23–24 on the next page.



Practice

Answers

23. Statements	Reasons
$\overline{AB} \parallel \overline{CD}$	Definition of parallelogram
$m\angle B + m\angle C = 180$ and $m\angle A + m\angle D = 180$	Same-Side Interior Angles Postulate
$\overline{BC} \parallel \overline{AD}$	Definition of parallelogram
$m\angle A + m\angle B = 180$ and $m\angle C + m\angle D = 180$	Same-Side Interior Angles Postulate
$m\angle A + m\angle B = m\angle B + m\angle C$	Trans. Prop. of Eq.
$m\angle A = m\angle C$	Subtr. Prop. of Eq.
$\angle A \cong \angle C$	Def. of congruence
$m\angle B + m\angle C = m\angle C + m\angle D$	Trans. Prop. of Eq.
$m\angle B = m\angle D$	Subtr. Prop. of Eq.
$\angle B \cong \angle D$	Def. of congruence

24. $m\angle 1 = 81$. In the parallelogram with the 99° angle, the adjacent angles are 81° by the Same-Side Interior Angles Postulate. In the parallelogram with $\angle 1$, the angle opposite $\angle 1$ is a vertical angle to one of the 81° angles, so the angle opposite $\angle 1$ is 81° by the Vertical Angles Theorem. By the Same-Side Interior Angles Postulate, the angles adjacent to $\angle 1$ must be 99° . Therefore, again, by the Same-Side Interior Angles Postulate, $m\angle 1 = 81$.
25. $y = 50$ because that angle and the labeled 50° angle are corresponding angles. The angles marked x and y are alternate interior angles, so $x = y = 50$.
26. $m\angle 1 = 61$ because it is a corresponding angle to $\angle 3$. Angles 1 and 2 are same-side interior angles, so they are supplementary. Therefore, $m\angle 2 = 180 - 61 = 119$.
27. a. 50°
b. corresponding angles; congruent
28. Minimum: 74° ; Maximum: 79°
29. Angles 3, 6, and 8 are congruent to angle 1; angles 4, 5, and 7 are congruent to angle 2.
30. 62

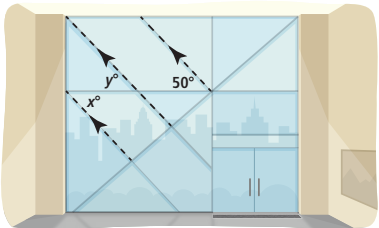


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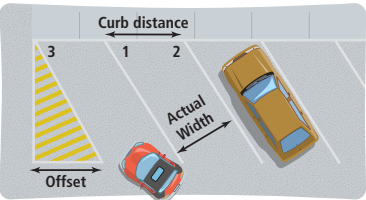
PRACTICE & PROBLEM SOLVING

APPLY

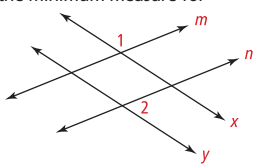
25. **Use a Problem Solving Model** A glazier is setting supports in parallel segments to prevent glass breakage during storms. What are the values of x and y ? Justify your conclusions.



26. **Evaluate Reasonableness** In the parking lot shown, all of the lines for the parking spaces should be parallel. If $m\angle 3 = 61$, what should $m\angle 1$ and $m\angle 2$ be? Explain.



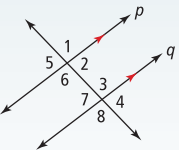
27. **Communicate Math Ideas** Margaret is in a boat traveling due west. She turned the boat 50° north of due west for a couple of minutes to get around a peninsula. Then she resumed due west again.
- a. How many degrees would she turn the wheel to resume a due west course?
- b. What type of angle pair did she use? Are the angles congruent or supplementary?
28. Parallel lines m and n intersect parallel lines x and y , representing two sets of intersecting railroad tracks. If the minimum measure for $\angle 1$ is 101° and the maximum measure for $\angle 1$ is 106° , what are the minimum and maximum measures for $\angle 2$?



ASSESSMENT PRACTICE

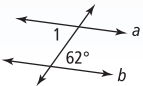
29. Classify each angle as congruent to $\angle 1$ or congruent to $\angle 2$. **G.5A**

Angle	Congruent to $\angle 1$	Congruent to $\angle 2$
$\angle 3$	☐	☐
$\angle 4$	☐	☐
$\angle 5$	☐	☐
$\angle 6$	☐	☐
$\angle 7$	☐	☐
$\angle 8$	☐	☐



30. **ACT** In the diagram, $a \parallel b$. What is $m\angle 1$?

- A** 28 **C** 90
B 62 **D** 118



31. **Performance Task** Students on a scavenger hunt are given the map shown and several clues. **G.6A**



Part A The first clue states the following:

Skyline Trail forms a transversal with Wood Path and Mission Path. Go to the corners that form same side exterior angles north of Skyline Trail.

Which two corners does the clue mean? Use intersections and directions to explain.

Part B The second clue states the following:

Wood and Mission Paths are parallel, and the northeast corner of Wood Path and Skyline Trail forms a 131° angle at trail marker M. Start at trail marker M and go to an equivalent exterior angle. Where are you now?

31. **Part A** the northwest corner of Skyline Trail and Mission Path and the northeast corner of Skyline Trail and Wood Path
Part B the southwest corner of Skyline Trail and Mission Path



QUICK CHECK

Use the Quick Check to assess students’ understanding of the mathematics in the lesson.

Students can take the Quick Check online where it will be automatically scored and appropriate differentiated practice will be assigned based on student performance. Or, you can download a printable copy from **SavvasRealize.com**. The Quick Check is also available in the *Assessment Sourcebook*.

Item Analysis

Item	DOK	Skills Review and Practice	Standards
1	2	G12	G.6A
2	3	G12	G.6A
3	3	G12	G.6A
4	3	G12	G.6A
5	2	G12	G.5A

DIFFERENTIATED RESOURCES

Points	Practice
0–3 points	- Reteach to Build Understanding - Mathematical Literacy and Vocabulary - Lesson Student Tutorial Videos
4–5 points	- Additional Practice - Enrichment

Quick Check is available at SavvasRealize.com.

ASSESSMENT RESOURCES

Name _____

SavvasRealize.com

2-1 Quick Check

Properties of Parallel Lines

1. Select all the true statements.

☐ A. $\angle 3 \cong \angle 2$ because they are alternate interior angles.

☒ B. $m\angle 1 + m\angle 3 = 180$ because they form a straight angle.

☒ C. $\angle 3 \cong \angle 6$ because they are alternate interior angles.

☒ D. $\angle 1$ and $\angle 6$ are supplementary because $\angle 3 \cong \angle 6$ and $m\angle 1 + m\angle 3 = 180$.

☐ E. $\angle 1 \cong \angle 3$ because they are vertical angles.

Use the figure shown for Items 2–4.

2. What is $m\angle 2 + m\angle 3$? Explain.

☐ A 180; $m\angle 1 + 90 = 180$, so $m\angle 2 + m\angle 3 = 180$.

☐ B 180; $m\angle 1 + 90 = m\angle 2 + m\angle 3$, so $m\angle 2 + m\angle 3 = 180$.

☒ C 90; $m\angle 1 + m\angle 2 + m\angle 3 = 180$. $m\angle 1 = 90$, so $m\angle 2 + m\angle 3 = 90$.

☐ D 90; $\angle 2$ and $\angle 3$ are corresponding angles, so $m\angle 2 + m\angle 3 = 90$.

3. If $m\angle 4 = 35$, find $m\angle 2$. Explain.

☐ A 55; $\angle 2$ and $\angle 4$ are complementary angles, so $m\angle 2 = 90 - m\angle 4$.

☐ B 35; $\angle 2$ and $\angle 4$ are corresponding angles, so $m\angle 2 = m\angle 4$.

☐ C 145; $\angle 2$ and $\angle 4$ are supplementary angles, so $m\angle 2 = 180 - m\angle 4$.

☒ D 35; $\angle 2$ and $\angle 4$ are alternate interior angles, so $m\angle 2 = m\angle 4$.

4. If $m\angle 4 = 35$, find $m\angle 3$. Explain.

☒ A 55; $\angle 3$ and $\angle 4$ are complementary angles, so $m\angle 3 = 90 - m\angle 4$.

☐ B 35; $\angle 3$ and $\angle 4$ are corresponding angles, so $m\angle 3 = m\angle 4$.

☐ C 145; $\angle 3$ and $\angle 4$ are supplementary angles, so $m\angle 3 = 180 - m\angle 4$.

☐ D 35; $\angle 3$ and $\angle 4$ are alternate interior angles, so $m\angle 3 = m\angle 4$.

5. In the figure shown, $t \parallel x$ and $k \parallel w$. If $m\angle 3 = 20$, choose the list which includes all of the other angles that measure 20° .

☐ A 1

☐ B 7, 9, 14

☐ C 1, 14, 16

☒ D 1, 5, 7, 9, 11, 14, 16

enVision+ Texas Geometry • Assessment Sourcebook

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Practice

DIFFERENTIATED RESOURCES

Reteach to Build Understanding

Provides scaffolded reteaching for the key lesson concepts.

Digital Version Online

2-1 Reteach to Build Understanding

Parallel Lines

1. Use the diagrams to fill in the blanks. Determine whether the pair of angles are congruent or supplementary and find the measure of the angle.

Same-Side Interior Angles: $\angle 1$ and $\angle 2$ are **supplementary**. If $m\angle 1 = 100^\circ$, then $m\angle 2 = 80^\circ$.

Alternate Interior Angles: $\angle 1$ and $\angle 2$ are **congruent**. If $m\angle 1 = 75^\circ$, then $m\angle 2 = 75^\circ$.

Corresponding Angles: $\angle 1$ and $\angle 2$ are **congruent**. If $m\angle 1 = 82^\circ$, then $m\angle 2 = 82^\circ$.

Alternate Exterior Angles: $\angle 1$ and $\angle 2$ are **congruent**. If $m\angle 1 = 77^\circ$, then $m\angle 2 = 77^\circ$.

2. A student said that $m\angle 1 = 80^\circ$. What error did the student likely make? What is $m\angle 1$?
The student likely confused same-side interior angles and alternate interior angles because they are both interior. $m\angle 1 = 100^\circ$.

3. Complete the two-column proof to prove the Alternate Exterior Angles Theorem.
Given: $k \parallel \ell$
Prove: $\angle 1 \cong \angle 7$

Statements	Reasons
1) $k \parallel \ell$	1) Given
2) $\angle 1 \cong \angle 5$	2) Corresponding Angles Theorem
3) $\angle 5 \cong \angle 7$	3) Vertical Angles Theorem
4) $\angle 1 \cong \angle 7$	4) Transitive Property of Congruence

Additional Practice

Provides extra practice for each lesson.

Digital Version Online

2-1 Additional Practice

Parallel Lines

Use the figure for Exercises 1-4. Identify all pairs of each type of angle.

- corresponding angles: $\angle 1$ and $\angle 5$; $\angle 2$ and $\angle 6$; $\angle 3$ and $\angle 7$; $\angle 4$ and $\angle 8$
- same-side interior angles: $\angle 2$ and $\angle 5$; $\angle 3$ and $\angle 6$
- alternate interior angles: $\angle 2$ and $\angle 8$; $\angle 3$ and $\angle 5$
- alternate exterior angles: $\angle 1$ and $\angle 7$; $\angle 4$ and $\angle 6$

Use the figure for Exercises 5 and 6.

5. Which angles are supplementary to the given angle?
 $\angle 1, \angle 2, \angle 4, \angle 6$

6. Which angles are congruent to the given angle?
 $\angle 3, \angle 5, \angle 7$

7. Complete the two-column proof.
Given: $x \parallel y$
Prove: $\angle 3 \cong \angle 5$

Statements	Reasons
1) $x \parallel y$	1) Given
2) $m\angle 3 + m\angle 8 = 180^\circ$	2) Same-Side Interior Angle Postulate
3) $m\angle 5 + m\angle 8 = 180^\circ$	3) Angle Addition Postulate
4) $m\angle 3 + m\angle 8 = m\angle 5 + m\angle 8$	4) Transitive Property of Equality
5) $m\angle 3 = m\angle 5$	5) Subtraction Property of Equality
6) $\angle 3 \cong \angle 5$	6) Definition of congruence

8. In the figure, $EH \parallel BI$ and $BI \parallel CJ$.

a. What is $m\angle 1$? Explain.
 127° ; by the Same-Side Interior Angles Postulate, $m\angle 1 + 53^\circ = 180^\circ$, so $m\angle 1 = 180^\circ - 53^\circ = 127^\circ$.

b. What is $m\angle 3$? Explain.
 53° ; by the Corresponding Angles Theorem, $\angle CMG \cong \angle 2$ and $\angle 2 \cong \angle 3$, so $\angle 3 \cong \angle CMG$ by the Transitive Property of Congruence, and $m\angle CMG = 53^\circ$, so $m\angle 3 = 53^\circ$.

Enrichment

Presents engaging problems and activities that extend the lesson concepts.

Digital Version Online

2-1 Enrichment

Parallel Lines

Mirrors can demonstrate some of the basic properties of parallel lines. Consider this diagram, in which $\overline{AB}$ represents the surface of the mirror.

A light ray is aimed at the mirror from C , strikes the mirror at O , and is reflected along OD . $\angle EOC$ is the angle of incidence, and $\angle EOD$ is the angle of reflection. The Law of Reflection states that the angle of incidence has the same measure as the angle of reflection.

1. Use the Law of Reflection to complete the two-column proof.
Given: $\angle AOC \cong \angle BOD$
Prove: $\overline{OE} \perp \overline{AB}$

Statements	Reasons
1) $\angle AOC \cong \angle BOD$	1) Given
2) $\angle COE \cong \angle DOE$	2) Law of Reflection
3) $m\angle AOC + m\angle COE = m\angle BOD + m\angle DOE$	3) Addition Property of Equality
4) $m\angle AOC + m\angle COE + m\angle BOD + m\angle DOE = 180^\circ$	4) Definition of a straight angle
5) $2(m\angle AOC + m\angle COE) = 180^\circ$	5) Substitution Property of Equality
6) $m\angle AOC + m\angle COE = 90^\circ$	6) Multiplication Property of Equality
7) $\overline{OE} \perp \overline{AB}$	7) Definition of perpendicular

You can construct a periscope with two parallel mirrors, as shown in the diagram. A ray of light enters from the object. It reflects off the first mirror and then the second mirror to the eye.

Use the diagram to answer these questions.

- What is true about $\angle 1$ and $\angle 2$? Explain.
 $\angle 1 \cong \angle 2$; Law of Reflection
- What is true about $\angle 2$ and $\angle 3$? Explain.
 $\angle 3 \cong \angle 2$; alternate interior angles
- What is true about $\angle 3$ and $\angle 4$? Explain.
 $\angle 3 \cong \angle 4$; Law of Reflection
- What can you conclude about $\angle 1$ and $\angle 4$? What property enables you to make that deduction?
 $\angle 1 \cong \angle 4$; substitution

Mathematical Literacy and Vocabulary

Helps emergent bilingual students develop and reinforce understanding of key terms and concepts.

2-1 Mathematical Literacy and Vocabulary

Parallel Lines

Complete the vocabulary table by filling in the missing information.

Word or Phrase	Definition	Figure or Example
angle pairs	Angle pairs are two angles that are related in some way.	congruent angles, supplementary angles, corresponding angles, exterior angles, interior angles, vertical angles
parallel lines	Two lines are parallel if they lie in the same plane and do not intersect.	
transversal	A transversal is a line that intersects two or more lines at distinct points.	
congruent	Two angles are congruent if the angles have the same measure.	
supplementary angles	The sum of the measures of two supplementary angles is 180° .	
alternate interior angles	Alternate interior angles are nonadjacent interior angles that lie on opposite sides of the transversal.	
corresponding angles	Corresponding angles are congruent angles that lie on the same side of the transversal and in corresponding positions.	

Expand the Learning

You may choose to provide time in class for students to work on the Pick a Project that they selected. Provide the Teacher Resource Originals found on SavvasRealize.com to students based on the project they chose. Consider allowing time for students to share their projects before the end of the Topic.

PICK A PROJECT

PROJECT 2A: Where do you see math in the real world? Create Your Own Math Walk

PROJECT 2B: Why are some roofs flat and some pitched? Plan a Roof

PROJECT 2C: What do parallel lines look like on a sphere? On a hemisphere? Explore Parallelism

PROJECT 2D: What makes optical illusion? Draw an Optical Illusion

TOPIC 2 Pick a Project 69